

ORTOGONALNI OKVIRI

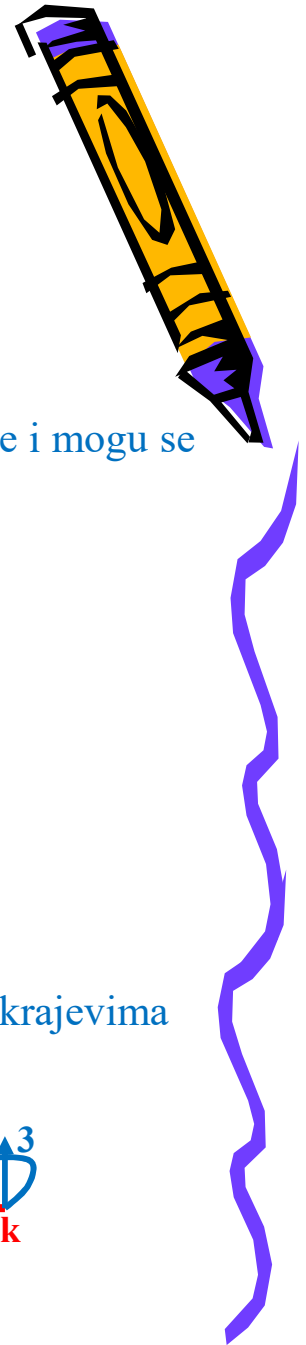
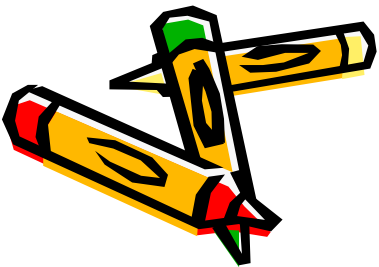
- Štapovi leže u jednom od dva međusobno ortogonalna pravca
- Konstrukcijski sistemi zgrada
- Aksijalne deformacije u odnosu na deformacije koje potiču od savijanja su neznatne i mogu se zanemariti

Dva pojednostavljena koja se uvode odnose se na:

- 1) uticaj normalnih sila na deformaciju nosača je zanemaren,*
- 2) izbor lokalnog koordinatnog sistema*

1) Uticaj normalnih sila na deformaciju nosača zanemaren

- smanjen broj stepeni slobode
- nepoznate u čvorovima su pomjeranja upravna na osu štapa i obrtanja na krajevima
- štap i-k ima 4 stepena slobode, po dva na krajevima



Matrica krutosti štapa i-k

$$S = \begin{bmatrix} M_i \\ M_k \end{bmatrix} = \begin{bmatrix} k_{22}^o & k_{23}^o \\ k_{32}^o & k_{33}^o \end{bmatrix} \begin{bmatrix} \tau_i \\ \tau_k \end{bmatrix} = \frac{1}{\alpha_{ik} \alpha_{ki} - \beta_{ik}^2} \begin{bmatrix} \alpha_{ik} & \beta_{ik} \\ \beta_{ik} & \alpha_{ki} \end{bmatrix} \begin{bmatrix} \tau_i \\ \tau_k \end{bmatrix}$$

$$R = \begin{bmatrix} T_i \\ M_i \\ T_k \\ M \end{bmatrix} = \begin{bmatrix} 1/1 & 1/1 \\ 1 & 0 \\ -1/1 & -1/1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M_i \\ M_k \end{bmatrix} = C^T S$$

$$\begin{bmatrix} \tau_i \\ \tau_k \end{bmatrix} = \begin{bmatrix} 1/1 & 1 & -1/1 & 0 \\ 1/1 & 0 & -1/1 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \varphi_i \\ v_k \\ \varphi \end{bmatrix} = Cq$$

$$a_{ik} = \frac{\alpha_{ki}}{\Delta}$$

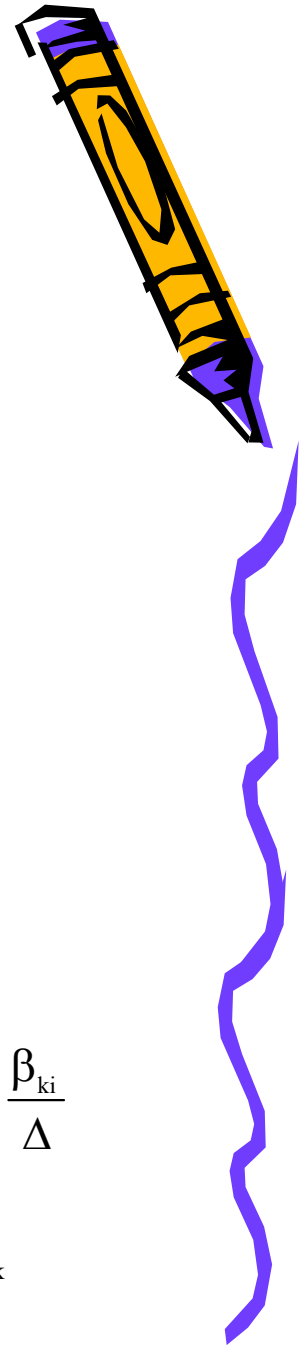
$$a_{ki} = \frac{\alpha_{ik}}{\Delta}$$

$$b_{ik} = b_{ki} = \frac{\beta_{ik}}{\Delta} = \frac{\beta_{ki}}{\Delta}$$

$$c_{ki} = a_{ki} + b_{ki}$$

$$c_{ik} = a_{ik} + b_{ik}$$

$$\Delta = \alpha_{ik} \alpha_{ki} - \beta_{ik}^2$$



$$\mathbf{k} = \mathbf{C}^T \mathbf{k}_o \mathbf{C} = \begin{bmatrix} 1/l & 1/l \\ 1 & 0 \\ -1/l & -1/l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{k}_{22}^o & \mathbf{k}_{23}^o \\ \mathbf{k}_{32}^o & \mathbf{k}_{33}^o \end{bmatrix} \begin{bmatrix} 1/l & 1 & -1/l & 0 \\ 1/l & 0 & -1/l & 1 \end{bmatrix}$$

$$\mathbf{k} = \begin{bmatrix} \frac{c_{ik} + c_{ki}}{l^2} & \frac{c_{ik}}{l} & -\frac{c_{ik} + c_{ki}}{l^2} & \frac{c_{ki}}{l} \\ \frac{c_{ik}}{l} & a_{ik} & -\frac{c_{ik}}{l} & b_{ik} \\ -\frac{c_{ik} + c_{ki}}{l^2} & -\frac{c_{ik}}{l} & \frac{c_{ik} + c_{ki}}{l^2} & -\frac{c_{ki}}{l} \\ \frac{c_{ki}}{l} & b_{ik} & -\frac{c_{ki}}{l} & a_{ki} \end{bmatrix}$$

EI = const.

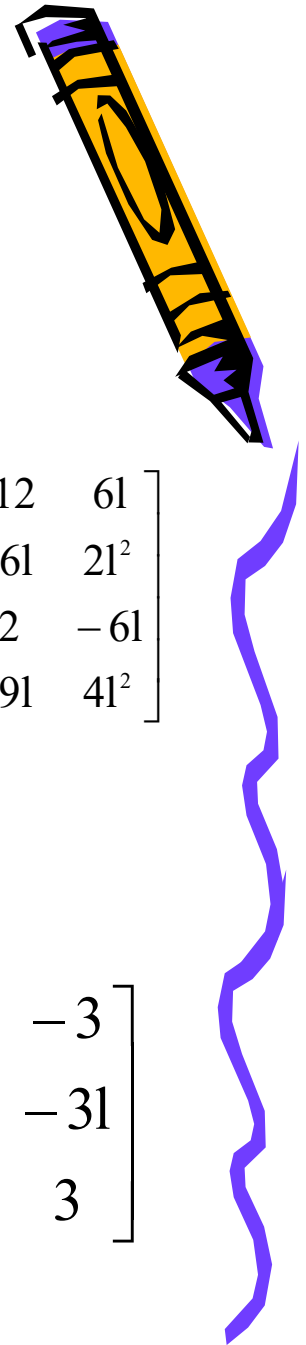
$$\mathbf{k} = \mathbf{C}^T \mathbf{k}_o \mathbf{C} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Matrica krutosti štapa i-g

$$\mathbf{k}_g = \mathbf{C}_g^T \mathbf{k}_{go} \mathbf{C}_g = \begin{bmatrix} \frac{d_{ig}}{l^2} & \frac{d_{ig}}{1} & -\frac{d_{ig}}{l^2} \\ \frac{d_{ig}}{1} & d_{ig} & -\frac{d_{ig}}{1} \\ -\frac{d_{ig}}{l^2} & -\frac{d_{ig}}{1} & \frac{d_{ig}}{l^2} \end{bmatrix}$$

EI = const.

$$\mathbf{k}_g = \frac{EI}{l^3} \begin{bmatrix} 3 & 3l & -3 \\ 3l & 3l^2 & -3l \\ -3 & -3l & 3 \end{bmatrix}$$



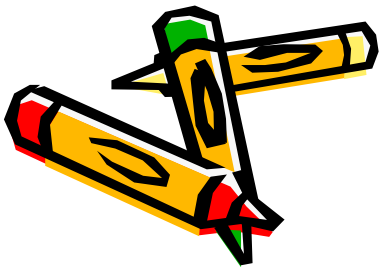
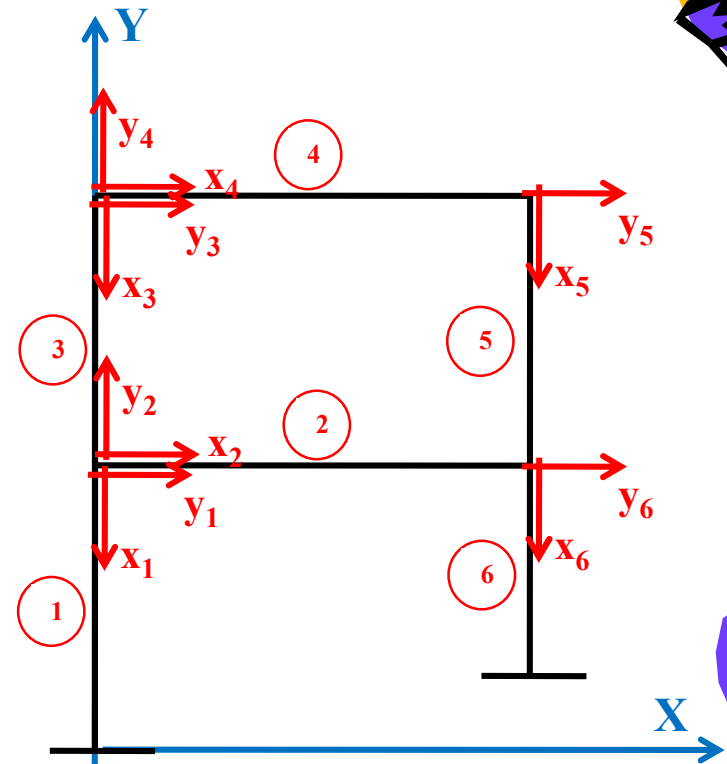
2) Drugo pojednostavljenje – izbor lokalnog koordinatnog sistema

- Transformacija iz lokalnog u globalni koordinatni sistem

Pogodnim izborom lokalnog koordinatnog sistema moguće je izbeći proces transformacije

- Lokalni koordinatni sistem **horizontalnih štapova** poklapa se sa globalnim koordinatnim sistemom, pr. štapovi 2 i 4 dati na slici

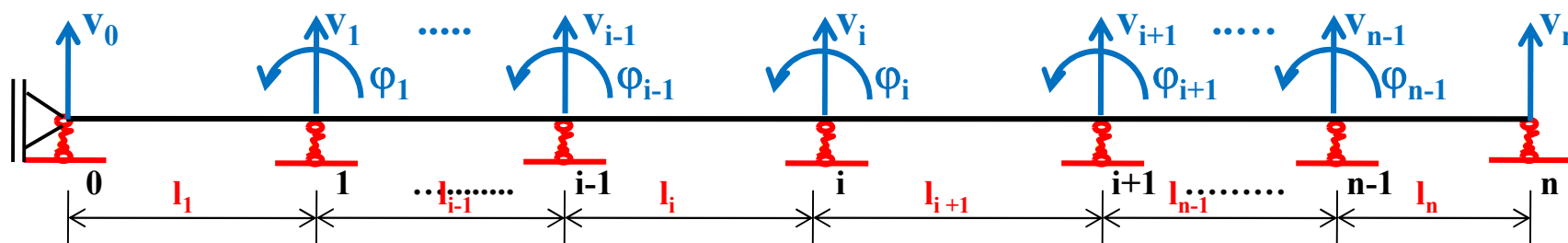
- Koordinatni početak lokalnog koordinatnog sistema **vertikalnih štapova** izabrati u čvoru na gornjem kraju štapa, štapovi 1,3,5 i 6 dati na slici:



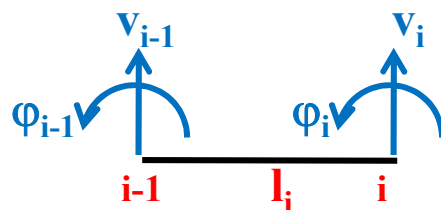
$$k_j^* = k_j = T_j^T k_j T_j \quad j = 1, 2, \dots, 6$$

KONTINUALNI NOSAČI

- Štap oslonjen na više od tri oslonca
- Jedan oslonac je nepokretan a svi ostali su pokretni
- Oslonci mogu biti elastični
- Specijalan slučaj ortogonalnih nosača
- Šema pomjeranja kao za ortogonalne okvire:



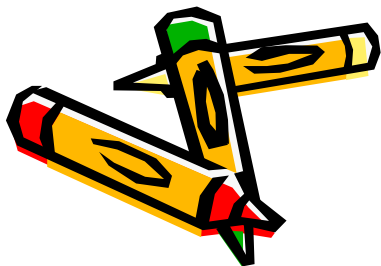
Za slučaj kada je $EI = \text{const}$



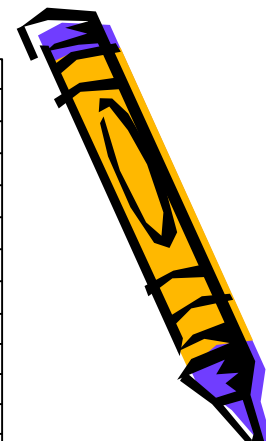
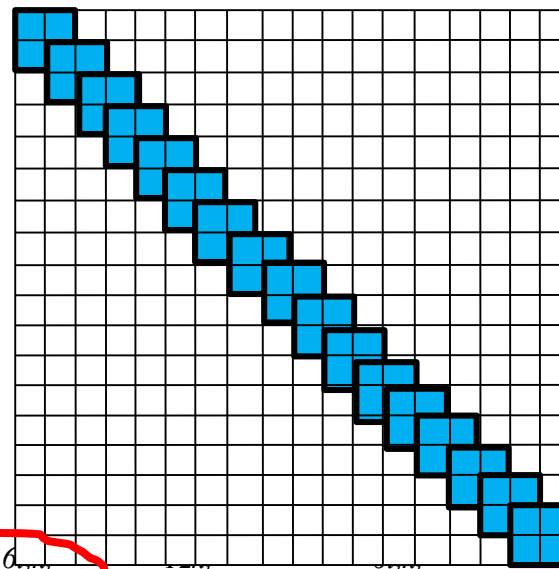
$$k_i = \frac{EI_i}{l_i^3}$$

$i = 2, \dots, n-1$

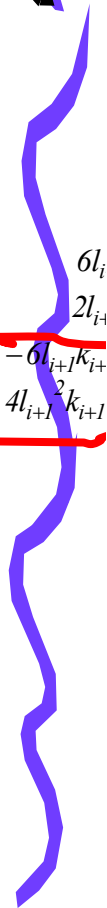
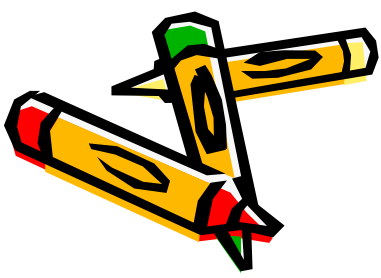
$$k = k_i \begin{bmatrix} 12 & 6l_i & -12 & 6l_i \\ 6l_i & 4l_i^2 & -6l_i & 2l_i^2 \\ -12 & -6l_i & 12 & -6l_i \\ 6l_i & 2l_i^2 & -6l_i & 4l_i^2 \end{bmatrix} = \begin{bmatrix} 12k_i & 6l_i k_i & -12k_i & 6l_i k_i \\ 6l_i k_i & 4l_i^2 k_i & -6l_i k_i & 2l_i^2 k_i \\ -12k_i & -6l_i k_i & 12k_i & -6l_i k_i \\ 6l_i k_i & 2l_i^2 k_i & -6l_i k_i & 4l_i^2 k_i \end{bmatrix}$$

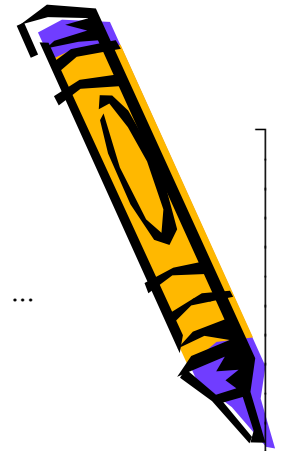


$C_i, i=1,2,3,\dots,n$ krutost opruge



$C_1 + 12k_1$	$-12k_1$	$6l_1k_1$
$-12k_1$	$C_2 + 12(k_1 + k_2)$	$-6l_1k_1 + 6l_2k_2$
$6l_1k_1$	$-6l_1k_1 + 6l_2k_2$	$4l_1^2k_1 + 4l_2^2k_2$
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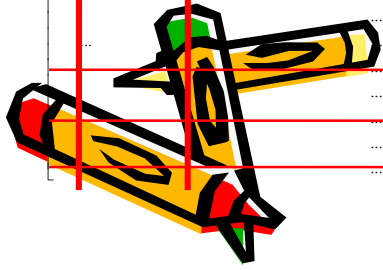




$$\begin{array}{cccccccc}
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 -12k_i & 6l_i k_i & & & & & & \\
 -6l_i k_i & 2l_i^2 k_i & & & & & & \\
 12(k_i + k_{i+1}) & -6l_i k_i + 6l_{i+1} k_{i+1} & -12k_{i+1} & 6l_{i+1} k_{i+1} & \dots & & & \\
 k_i + 6l_{i+1} k_{i+1} & 4l_i^2 k_i + 4l_{i+1}^2 k_{i+1} & -6l_{i+1} k_{i+1} & 2l_{i+1}^2 k_{i+1} & \dots & & & \\
 -12k_{i+1} & -6l_{i+1} k_{i+1} & C_{i+1} + 12(k_{i+1} + k_{i+2}) & -6l_{i+1} k_{i+1} + 6l_{i+2} k_{i+2} & \dots & & & \\
 6l_{i+1} k_{i+1} & 2l_{i+1}^2 k_{i+1} & -6l_{i+1} k_{i+1} + 6l_{i+2} k_{i+2} & 4l_{i+1}^2 k_{i+1} + 4l_{i+2}^2 k_{i+2} & \dots & & & \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & C_{n-2} + 12(k_{n-2} + k_{n-1}) & 6l_i k_i & -12k_i & 6l_i k_i & & & \\
 \dots & 6l_i k_i & 4l_i^2 k_i & -6l_i k_i & 2l_i^2 k_i & & & \\
 \dots & -12k_i & -6l_i k_i & C_{n-1} + 12(k_{n-1} + k_n) & -6l_{n-1} k_{n-1} + 6l_n k_n & -12k_n & & \\
 \dots & 6l_i k_i & 2l_i^2 k_i & -6l_{n-1} k_{n-1} + 6l_n k_n & 4l_{n-1}^2 k_{n-1} + 4l_n^2 k_n & -6l_n k_n & & \\
 \dots & \dots & \dots & -12k_n & -6l_n k_n & C_n + 12k_n & &
 \end{array}$$

$$\begin{array}{cccc}
 \dots & C_{n-2} + 12(k_{n-2} + k_{n-1}) & 6l_i k_i & -12k_i & 6l_i k_i \\
 & 6l_i k_i & 4l_i^2 k_i & -6l_i k_i & 2l_i^2 k_i \\
 \dots & -12k_i & -6l_i k_i & C_{n-1} + 12(k_{n-1} + k_n) & -6l_{n-1} k_{n-1} + 6l_n k_n & -12k_n \\
 \dots & 6l_i k_i & 2l_i^2 k_i & -6l_{n-1} k_{n-1} + 6l_n k_n & 4l_{n-1}^2 k_{n-1} + 4l_n^2 k_n & -6l_n k_n \\
 \dots & \dots & \dots & -12k_n & -6l_n k_n & C_n + 12k_n
 \end{array}$$

$$\begin{array}{cccccccc}
 C_1 + 12k_1 & -12k_1 & 6l_1 k_1 & & & & & \\
 -12k_1 & C_2 + 12(k_1 + k_2) & -6l_1 k_1 + 6l_2 k_2 & \dots & & & & \\
 6l_1 k_1 & -6l_1 k_1 + 6l_2 k_2 & 4l_1^2 k_1 + 4l_2^2 k_2 & \dots & & & & \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\
 \dots & C_{i-1} + 12(k_{i-1} + k_i) & -6l_{i-1} k_{i-1} + 6l_i k_i & -12k_i & 6l_i k_i & \dots & \dots & \\
 \dots & -6l_{i-1} k_{i-1} + 6l_i k_i & 4l_{i-1}^2 k_{i-1} + 4l_i^2 k_i & -6l_i k_i & 2l_i^2 k_i & \dots & \dots & \\
 \dots & -12k_i & -6l_i k_i & C_i + 12(k_i + k_{i+1}) & -6l_i k_i + 6l_{i+1} k_{i+1} & -12k_{i+1} & 6l_{i+1} k_{i+1} & \\
 k = & 6l_i k_i & 2l_i^2 k_i & -6l_i k_i + 6l_{i+1} k_{i+1} & 4l_i^2 k_i + 4l_{i+1}^2 k_{i+1} & -6l_{i+1} k_{i+1} & 2l_{i+1}^2 k_{i+1} & \\
 \dots & \dots & \dots & -12k_{i+1} & -6l_{i+1} k_{i+1} & C_{i+1} + 12(k_{i+1} + k_{i+2}) & -6l_{i+1} k_{i+1} + 6l_{i+2} k_{i+2} & \\
 \dots & \dots & \dots & 6l_{i+1} k_{i+1} & 2l_{i+1}^2 k_{i+1} & -6l_{i+1} k_{i+1} + 6l_{i+2} k_{i+2} & 4l_{i+1}^2 k_{i+1} + 4l_{i+2}^2 k_{i+2} & \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\
 \dots & C_{n-2} + 12(k_{n-2} + k_{n-1}) & 6l_i k_i & -12k_i & 6l_i k_i & & & \\
 \dots & 6l_i k_i & 4l_i^2 k_i & -6l_i k_i & 2l_i^2 k_i & & & \\
 \dots & -12k_i & -6l_i k_i & C_{n-1} + 12(k_{n-1} + k_n) & -6l_{n-1} k_{n-1} + 6l_n k_n & -12k_n & & \\
 \dots & 6l_i k_i & 2l_i^2 k_i & -6l_{n-1} k_{n-1} + 6l_n k_n & 4l_{n-1}^2 k_{n-1} + 4l_n^2 k_n & -6l_n k_n & & \\
 \dots & \dots & \dots & -12k_n & -6l_n k_n & C_n + 12k_n & &
 \end{array}$$



Za nosače sa nepomjerljivim čvorovima:

$$v_i=0, i=1,2,3,\dots$$

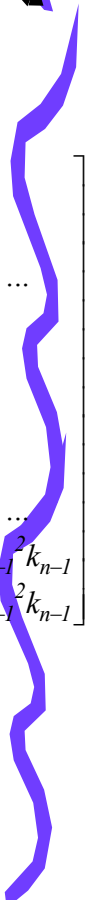
Nepoznata pomjeranja su:

$$\varphi_i, i=1,2,3,\dots$$

$$k = \begin{bmatrix} 12k_1 & 6l_1k_1 & -12k_1 & 6l_1k_1 \\ 6l_1k_1 & 4l_1^2k_1 & -6l_1k_1 & 2l_1^2k_1 \\ -12k_1 & -6l_1k_1 & 12k_1 & -6l_1k_1 \\ 6l_1k_1 & 2l_1^2k_1 & -6l_1k_1 & 4l_1^2k_1 \end{bmatrix}$$

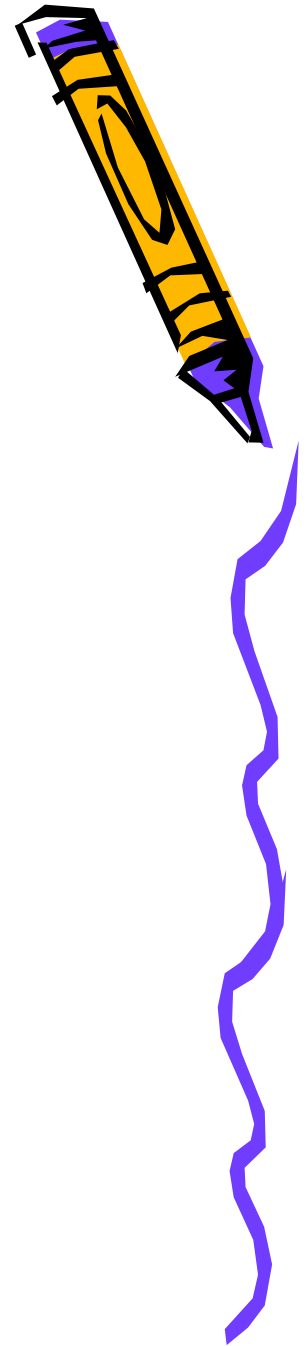
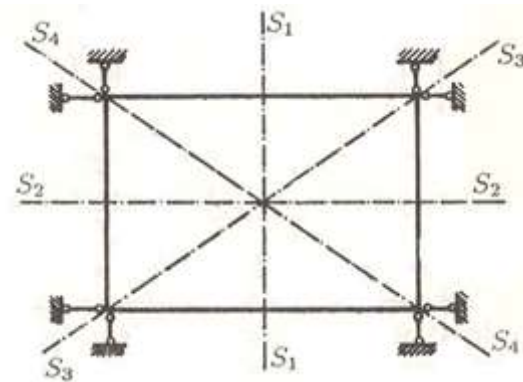
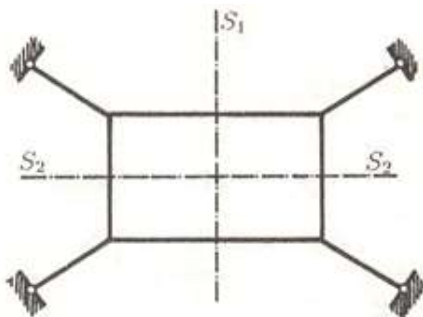
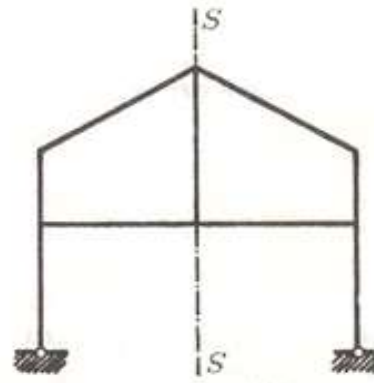
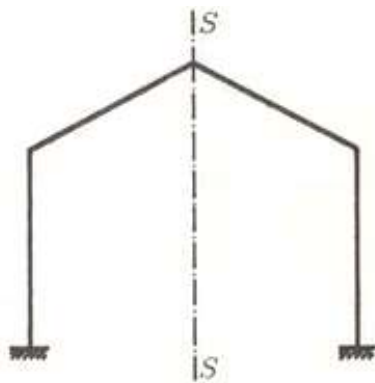


$$K^* = \begin{bmatrix} 4l_1^2k_1 + 4l_2^2k_2 & 2l_2^2k_2 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 2l_2^2k_2 & 4l_2^2k_2 + 4l_3^2k_3 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 4l_{i-1}^2k_{i-2} + 4l_i^2k_{i-1} & 2l_i^2k_i & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 2l_i^2k_i & 4l_i^2k_i + 4l_{i+1}^2k_{i+1} & 2l_{i+1}^2k_{i+1} & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 2l_{i+1}^2k_{i+1} & 4l_i^2k_i + 4l_{i+1}^2k_{i+1} & 2l_{i+2}^2k_{i+2} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 2l_{i+2}^2k_{i+2} & 4l_{i+1}^2k_{i+1} + 4l_{i+2}^2k_{i+2} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 4l_{n-2}^2k_{n-2} + 4l_{n-1}^2k_{n-1} & 2l_{n-1}^2k_{n-1} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 2l_{n-1}^2k_{n-1} & 4l_{n-1}^2k_{n-1} & \dots \end{bmatrix}$$



SIMETRIČNI NOSAČI

- Svi elementi raspoređeni su simetrično u odnosu na jednu ili više osa simetrije
- Simetrično postavljene štapove imaju iste geometrijske i mehaničke karakteristike
- Konturni uslovi, uslovi oslanjanja, su simetrični



Proizvoljno opterećenje se razlaže na:

- Simetrično opterećenje
- Antimetrično opterećenje

Princip superpozicije uticaja

- Simetrično opterećenje izaziva simetrične uticaje
- Antimetrično opterećenje izaziva antimetrične uticaje

IZBOR GENERALISANIH POMJERANJA

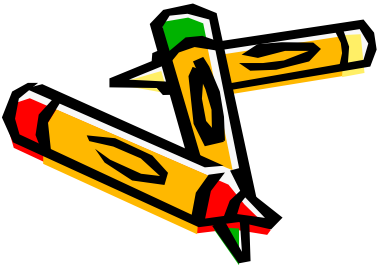
Generalisana pomjeranja u čvorovima simetričnih nosača možemo razdvojiti na simetrična i antimetrična.

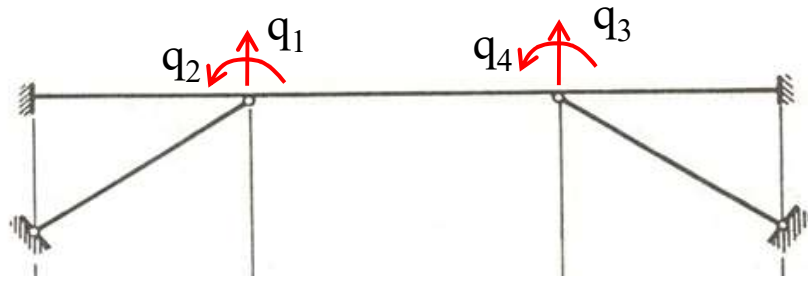
\mathbf{r}^* vektor generalisanih pomjeranjima koja su nastala usled simetričnog i antimetričnog opterećenja

Članovi tog vektora su generalisana pomjeranja \mathbf{r}_i **VODI SE RAČUNA O SIMETRIJI**

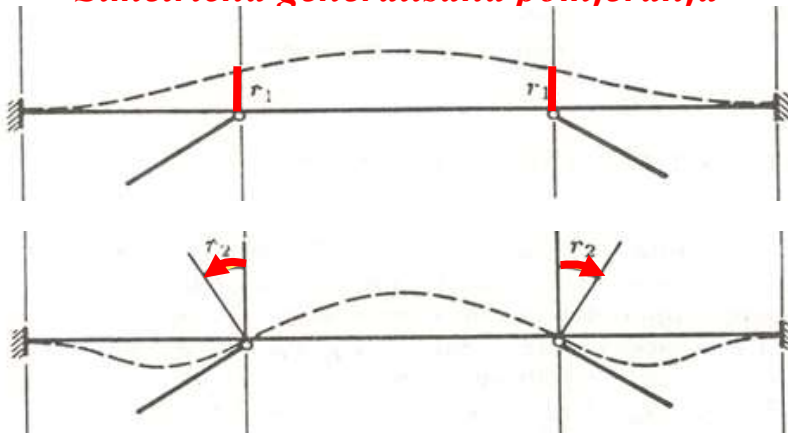
\mathbf{q}^* vektor generalisanih pomjeranjima zadatog simetričnog nosača usled zadatog opterećenja..

Članovi tog vektora su generalisana pomjeranja \mathbf{q}_i – **NE VODI SE RAČUNA O SIMETRIJI**





Simetrična generalisana pomjeranja



$$q_1 = r_1 + r_3$$

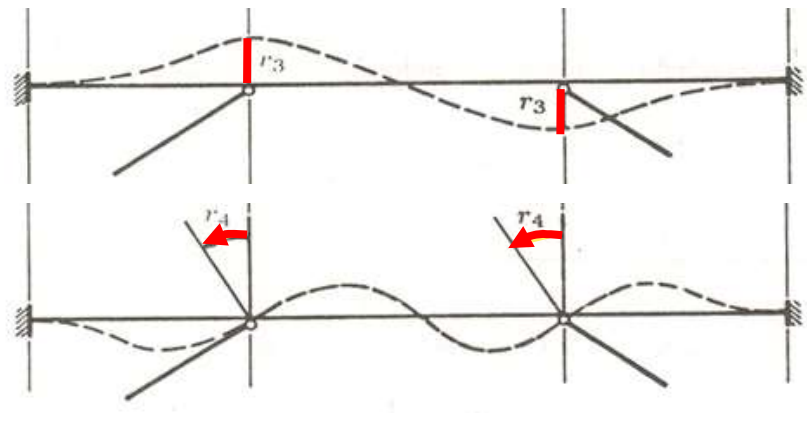
$$q_2 = r_2 + r_4$$

$$q_3 = r_1 - r_3$$

$$q_4 = -r_2 + r_4$$

+

Antimetrična generalisana pomjeranja



$$q^* = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}$$

$$q^* = \hat{T}r^*$$



Generalisanim pomjeranjima q^* odgovaraju

K_q^* matrica krutosti

Q_q^* vektor ekvivalentnog opterećenja

R_q^* vektor generalisanih sila

Generalisanim pomjeranjima r^* odgovaraju

K_r^* matrica krutosti

Q_r^* vektor ekvivalentnog opterećenja

R_r^* vektor generalisanih sila

$$R_r^* = K_r^* r^*$$

$$R_q^* = K_q^* q^*$$

$$R_q^* = K_q^* \hat{T} r^*$$

$$\hat{T}^T R_q^* = \hat{T}^T K_q^* \hat{T} r^*$$

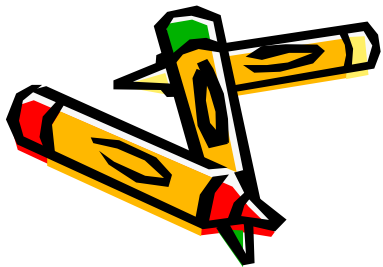
$$K_r^* = \hat{T}^T K_q^* \hat{T}$$

$$q^* = \hat{T} r^*$$

$$R_r^* = \hat{T}^T R_q^*$$

$\times \hat{T}^T$ sa lijeve strane

$$R_r^* = \hat{T}^T K_q^* \hat{T} r^*$$



Sistem jednačina

$$\begin{bmatrix} \mathbf{K}_s^* & \\ & \mathbf{K}_a^* \end{bmatrix} \begin{bmatrix} \mathbf{r}_s^* \\ \mathbf{r}_a^* \end{bmatrix} = \begin{bmatrix} \mathbf{S}_s^* \\ \mathbf{S}_a^* \end{bmatrix}$$

Matrica \mathbf{K}_r^* kvazidijagonalna matrica koja se sastoji od dva dijagonalna bloka

\mathbf{K}_s za simetriju

\mathbf{K}_a za antimetriju

Sistem jednačina iz kojih se **određuju simetrična \mathbf{r}_s^* i antimetrična \mathbf{r}_a^* generalisana pomjeranja**

$$\mathbf{K}_s^* \mathbf{r}_s^* = \mathbf{S}_s^*$$

$$\mathbf{K}_a^* \mathbf{r}_a^* = \mathbf{S}_a^* \quad \Rightarrow \mathbf{r}^*$$

Nakon toga se određuje vektora \mathbf{q}^* i sile u štapovima:

$$\mathbf{q}^* = \hat{T} \mathbf{r}^*$$

$$R_j = k_j T_j q_j^* - Q_j$$



KONTURNI USLOVI U OSI SIMETRIJE

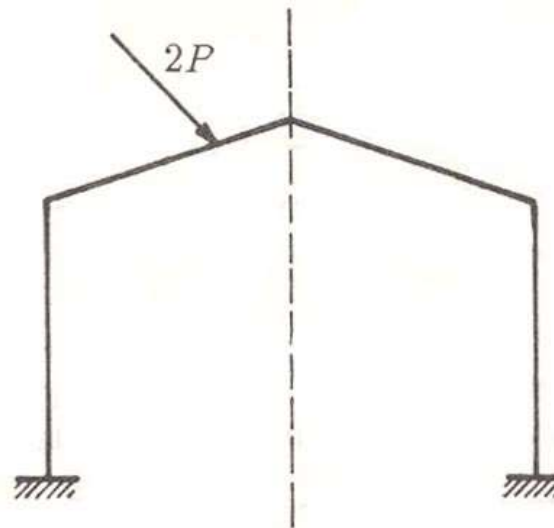
Statičko-deformacijska analiza simetričnih nosača:

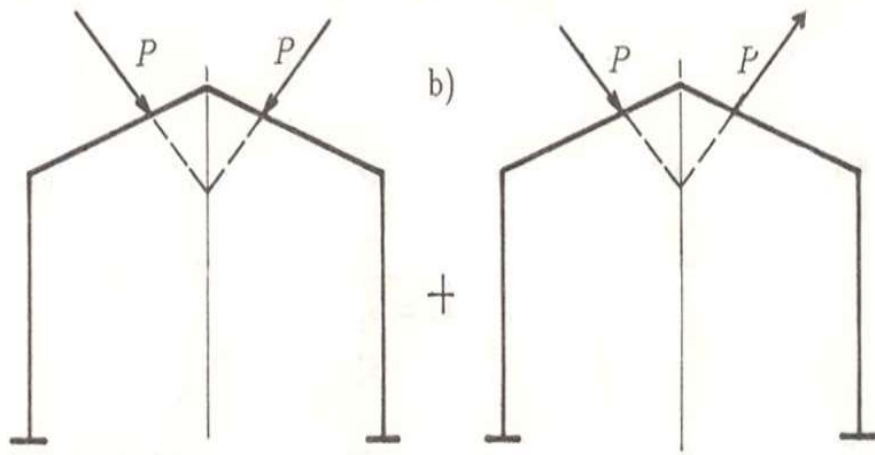
- Usled dejstva simetričnog opterećenja
- Usled dejstva antimetričnog opterećenja

Matrica \hat{T} je često velikog reda pa je pogodnije razmatrati polovinu nosača

Konturni uslovi se definišu posebno za slučaj antimetričnog opterećenja a posebno za slučaj simetričnog opterećenja

1) osa simetrije prolazi kroz čvor



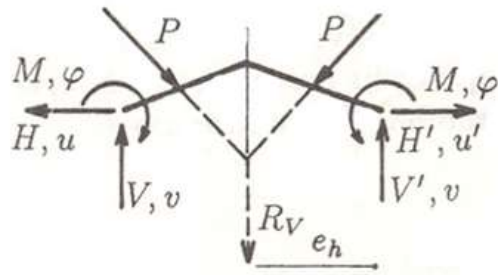


Simetrija:

$$H = H'$$

$$V + V' = R_v$$

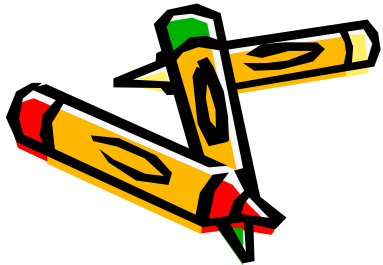
$$M = M'$$



uslovi u osi simetrije

$$H \neq 0 \quad V = 0 \quad M \neq 0$$

$$u = 0 \quad v \neq 0 \quad \varphi = 0$$

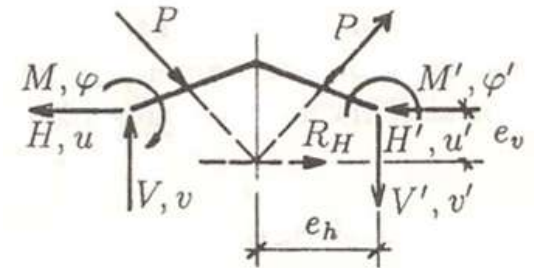


Antimetrija:

$$H + H' = R_h$$

$$V = V'$$

$$M + M' + (V + V')e_h = R_h e_v$$



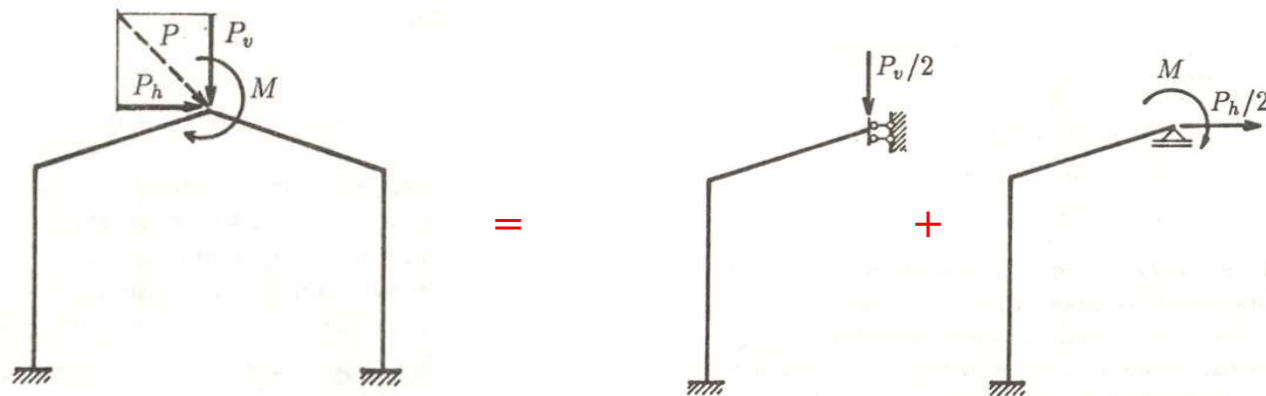
uslovi u osi simetrije

$$H = 0 \quad V \neq 0 \quad M = 0$$

$$u \neq 0 \quad v = 0 \quad \varphi \neq 0$$

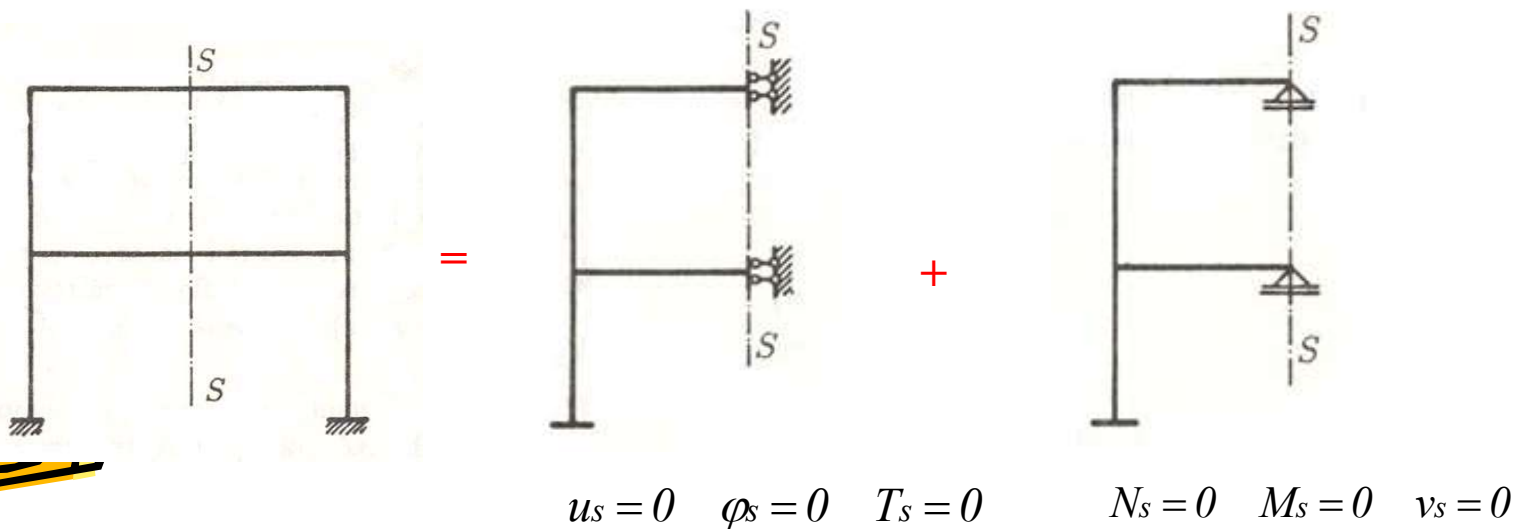


Razlaganje opterećenja koje djeluje u osi simetrije:



Matrice krutosti i vektor ekvivalentnog opterećenja određuje se na način koji je izložen.

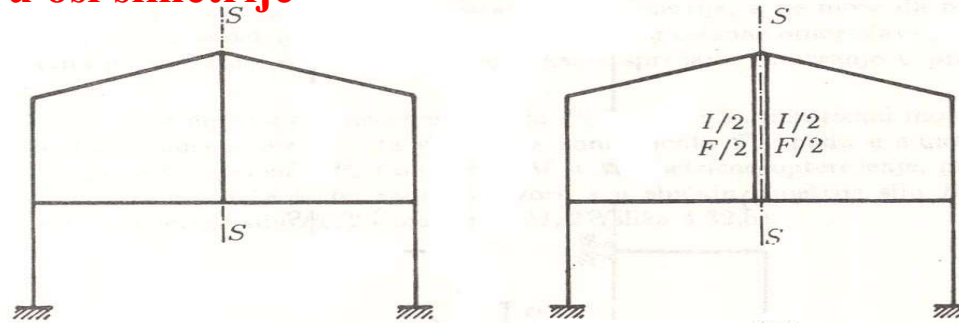
2) osa simetrije siječe štapove nosača-polovi horizontalne štapove sistema



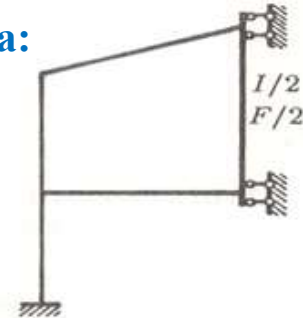
Matrice krutosti i vektor ekvivalentnog opterećenja za antimetrično opterećenu polovinu nosača određuje se na način koji je izložen.

Kod simetrično opterećene polovine nosača definišu se štapovi tipa "s" za koje je potrebno definisati matricu krutosti i vektor ekvivalentnog opterećenja.

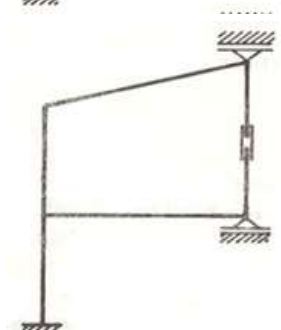
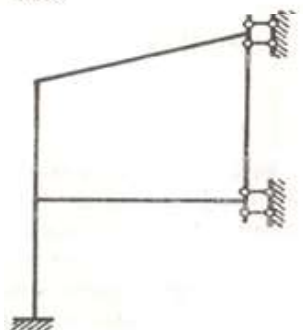
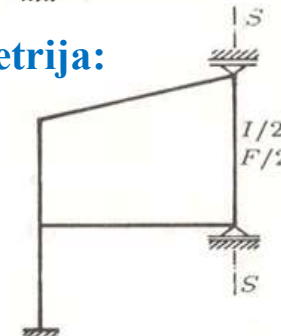
3) nosač sa štapovima u osi simetrije

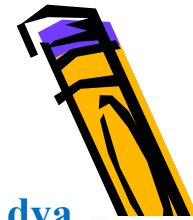


Simetrija:



Antimetrija:



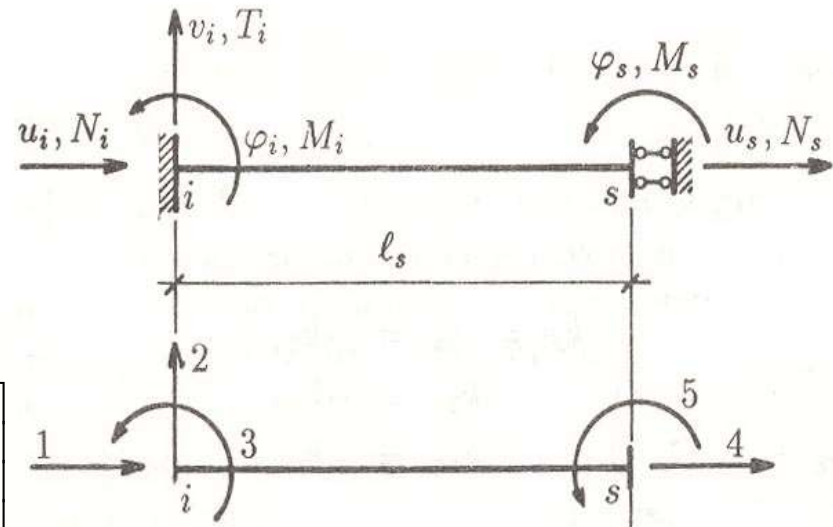


Matrica krutosti štapa tipa "s"

- Štap tipa "s" je štap koga osa simetrije siječe pod uglom od 90° i dijeli ga na dva jednaka dijela.
- $T_s=0$
- 5 generalisanih pomjeranja štapa "s"

Polazi se od poznate veze:

$$\begin{bmatrix} N_i \\ T_i \\ M_i \\ N_s \\ T_s \\ M_s \end{bmatrix} = \begin{bmatrix} k_{11} & & & & & \\ & k_{22} & k_{23} & & & \\ & k_{32} & k_{33} & & & \\ k_{41} & & & k_{44} & & \\ & k_{52} & k_{53} & & k_{55} & k_{56} \\ & k_{62} & k_{63} & & k_{65} & k_{66} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ \varphi_i \\ u_s \\ v_s \\ \varphi_s \end{bmatrix}$$



uz uslov $T_s=0$

$$v_s = \frac{-1}{k_{55}} (k_{52}v_i + k_{53}\varphi_i + k_{56}\varphi_s)$$



$$\begin{bmatrix} N_i \\ T_i \\ M_i \\ N_s \\ M_s \end{bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 & k_{14} & 0 \\ & k_{22} - \frac{k_{25}^2}{k_{55}} & k_{23} - \frac{k_{25}k_{53}}{k_{55}} & 0 & k_{26} - \frac{k_{25}k_{56}}{k_{55}} \\ & & k_{33} - \frac{k_{35}^2}{k_{55}} & 0 & k_{36} - \frac{k_{35}k_{56}}{k_{55}} \\ & \text{simetrič} & & k_{44} & 0 \\ & & & & k_{66} - \frac{k_{56}^2}{k_{55}} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ \varphi_i \\ u_s \\ \varphi_s \end{bmatrix}$$

$$R_s = k_s q_s$$

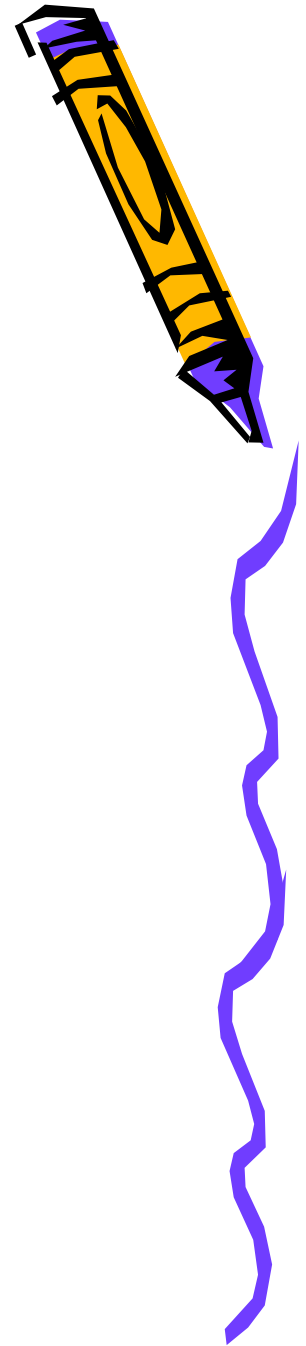
Zbog simetrije u odnosu na glavnu dijagonalu:

$$k = \begin{bmatrix} \frac{EF}{l} & 0 & 0 & -\frac{EF}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{EF}{l} & 0 & 0 & \frac{EF}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix}$$

$$k_{22} = k_{55} = -k_{25}$$

$$k_{23} = -k_{35}$$

$$k_{26} = -k_{56}$$



Matrica štapa s postaje:

$$k_s = \begin{bmatrix} k_{11} & 0 & 0 & k_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{33} - \frac{k_{35}^2}{k_{55}} & 0 & k_{36} - \frac{k_{35}k_{56}}{k_{55}} \\ k_{14} & 0 & 0 & k_{44} & 0 \\ 0 & 0 & k_{36} - \frac{k_{35}k_{56}}{k_{55}} & 0 & k_{66} - \frac{k_{56}^2}{k_{55}} \end{bmatrix}$$

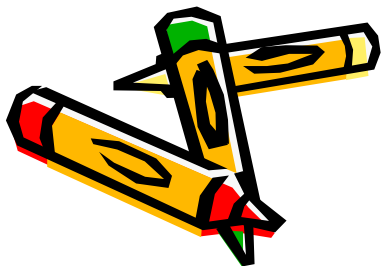
Za $EI = \text{const}$

$$k_{33} = k_{66} = \frac{4EI}{l_s}$$

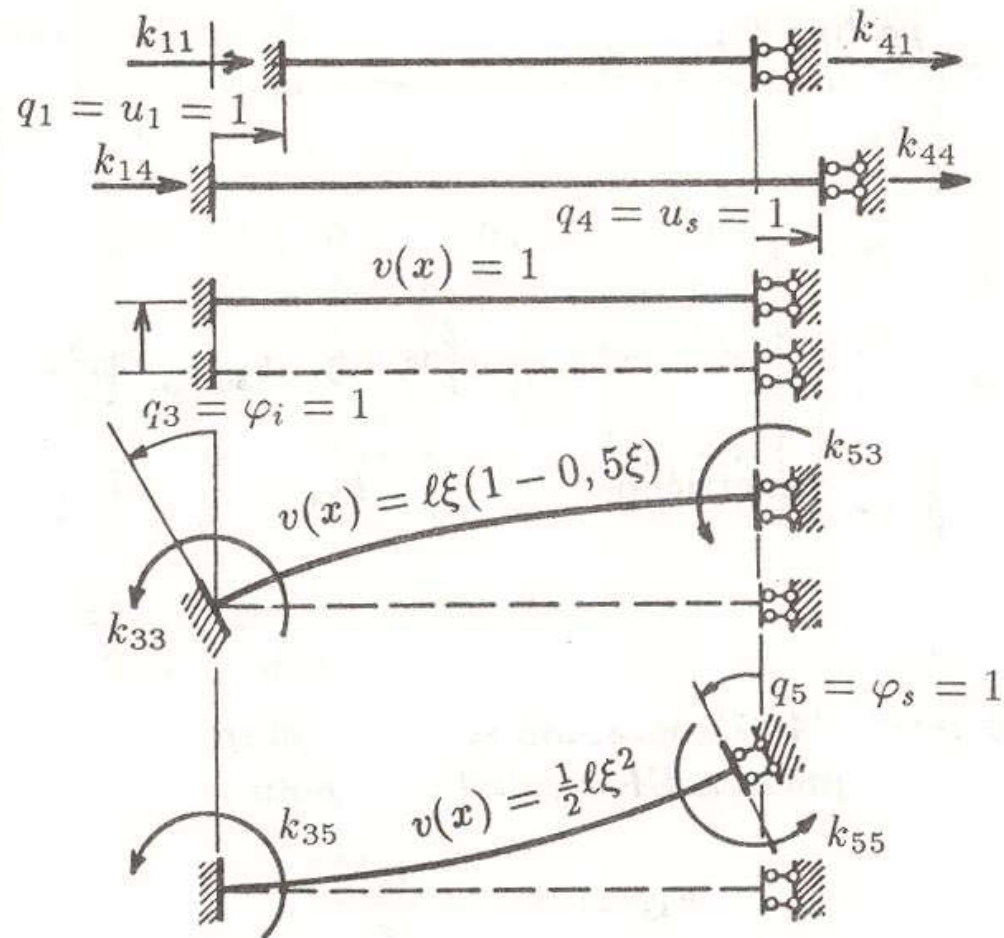
$$k_{35} = k_{56} = -\frac{6EI}{l_s^2}$$

$$k_{55} = \frac{12EI}{l_s^3}$$

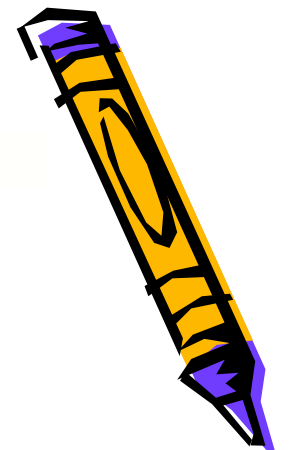
$$k = \begin{bmatrix} \frac{EF}{l} & 0 & 0 & -\frac{EF}{l} & 0 & 0 \\ 0 & \frac{12EI l_s}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI l_s}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{EF}{l} & 0 & 0 & \frac{EF l_s}{l} & 0 & 0 \\ 0 & -\frac{12EI l_s}{l^3} & -\frac{6EI}{l^2} & 0 & \frac{12EI l_s}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ EI \\ l_s \\ 0 \\ EI \end{bmatrix}$$



Geometrijsko-statičko značenje:

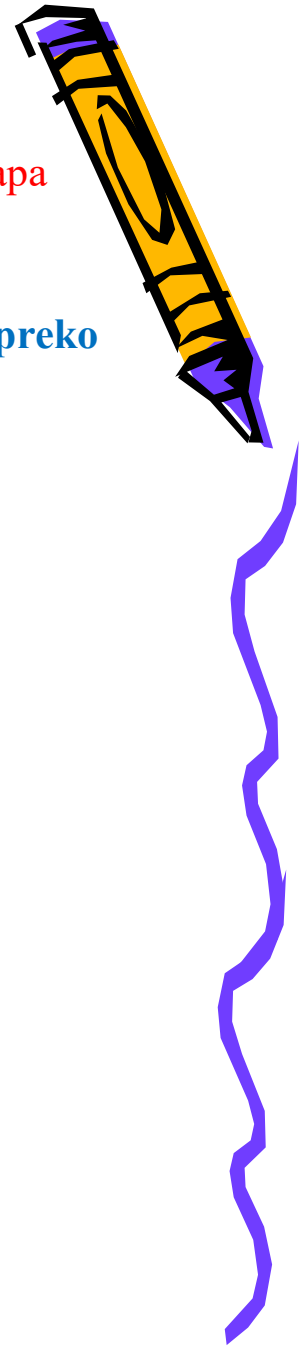
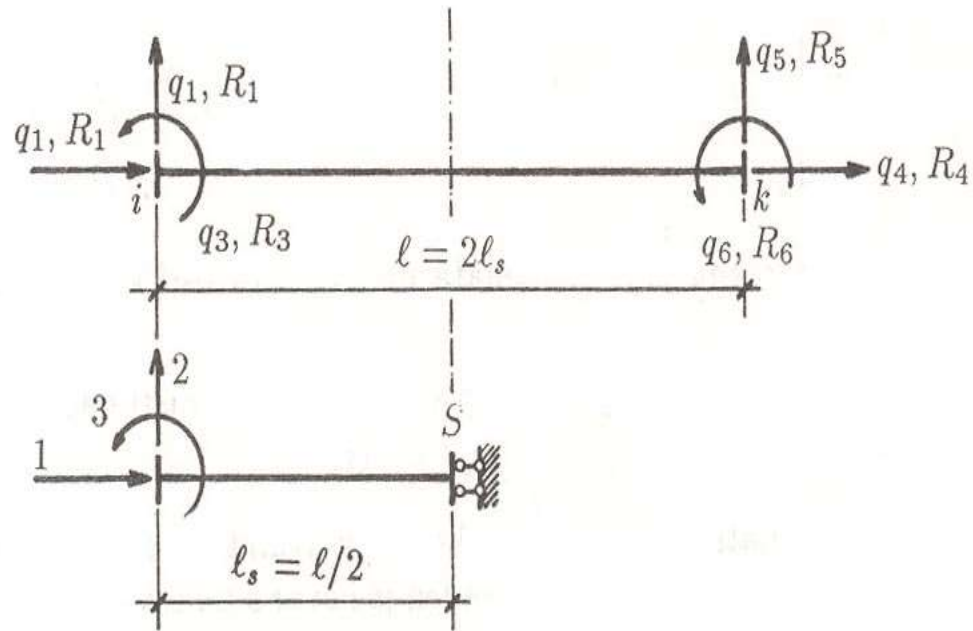


Vektor ekvivalentnog čvornog opterećenja za štap tipa s određuje se na isti način kao i za štapove tipa k, odnosno g.



Nije potrebno uvoditi čvor s jer se u tom čvoru uticaji mogu odrediti analizom štapa

Sile i pomjeranja u osi simetrije za štapove tipa s mogu da se eliminišu i izraze preko sila i pomjeranja na krajevima štapa, kao što je prikazano na slici:



Zbog simetrije:

$$\begin{aligned} q_4 &= -q_1 & R_4 &= -R_1 \\ q_5 &= q_2 & R_5 &= R_2 \\ q_6 &= -q_3 & R_6 &= -R_3 \end{aligned} \quad \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} k_{11} - k_{14} & 0 & 0 \\ 0 & k_{22} + k_{25} & k_{23} - k_{26} \\ 0 & k_{32} + k_{35} & k_{33} - k_{36} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

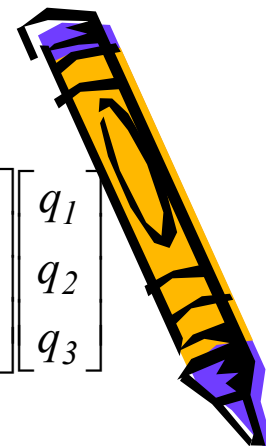
$$\begin{aligned} k_{22} &= -k_{25} & k_{11} &= -k_{14} = k_{11}^0 \\ k_{23} &= k_{26} & k_{22} &= k_{55} = k_{22}^0 \\ k_{32} &= -k_{35} & k_{36} &= k_{23}^0 \end{aligned} \quad \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 2k_{11}^0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k_{22}^0 - k_{23}^0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

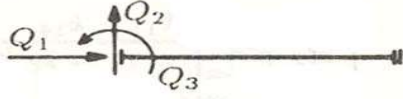
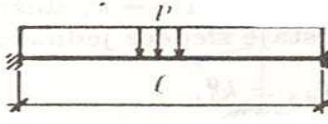
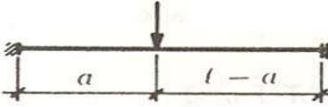
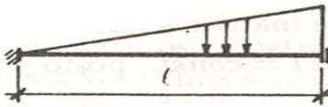
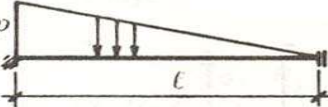
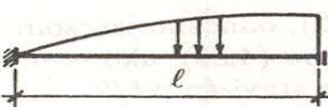
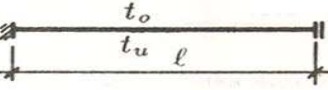
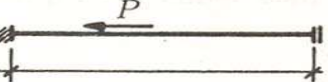
Koeficijenti bazne matrice:

$$k = \begin{bmatrix} \frac{EF}{l} & 0 & 0 & -\frac{EF}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{EF}{l} & 0 & 0 & \frac{EF}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix}$$

$$k_{11}^0 = \frac{EF}{l} \quad k_{22}^0 = \frac{4EI}{l} \quad k_{23}^0 = \frac{2EI}{l}$$

$$k_s = \begin{bmatrix} \frac{2EF}{l} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{2EI}{l} \end{bmatrix} \quad l_s = \frac{l}{2}$$



opterećenje	
	$Q_2 = -p\ell$ $Q_3 = -\frac{p\ell^2}{3}$
	$Q_2 = -P$ $Q_3 = -Pa\left(1 - \frac{a}{2\ell}\right)$
	$Q_2 = \frac{p\ell}{2}$ $Q_3 = -\frac{p\ell^2}{24}$
	$Q_2 = -\frac{p\ell}{2}$ $Q_3 = -\frac{p\ell^2}{8}$
	$Q_2 = -\frac{2p\ell}{3}$ $Q_3 = -\frac{2p\ell^2}{15}$
	$Q_1 = -\frac{\alpha EF}{2}(t_0 - t_u), \quad Q_2 = 0$ $Q_3 = EI\alpha \frac{t_0 - t_u}{h}$
	$Q_1 = P$

